Debt Sustainability Condition ±Mathematical Version

The famous debt sustainability condition is often written mathematically as follows:

¿ l nš / l š··š p L :˜š F •š; l nš?Ú / l š?Ú··š?Ú p E sš F €š / l š··š F ¿t š / l š··š

SIMPLE ANALYTICS:

Public Debt-to-GDP ratio will rise if:

˜š P •š

sš F €š P Ù

Debt sustainability requires:

¿ l nš / l š··š p L Ù

If debt monetization is pursued ¿*ç P r, it will help reduce the debt burden. However, debt monetization may result in higher inflation. If we rule out debt monetization (¿*ç L r), then debt sustainability requires:

F l sš F €š / l š··š p L :˜š F •š; l nš?Ú / l š?Ú··š?Ú p

or, !0ˆ^ y.ìÍ å y.¿1/ „˜CW •, à Kä a''ëó ñ l„5qp R0 ÄÓ a''ëó ñ l„5qp p† à >@à y.¿1/ •“ à

l €š F sš / l š··š p L :˜š F •š; l nš?Ú / l š?Ú··š?

)ç: Government Purchases

6ç: Taxes net of Transfers (6ç L 6 = T A F 6 N = J O) A N O

)ç F 6ç: Primary Budget Deficit (note:)ç P 6ç implies a primary budget deficit)

$ç ã Government Debt in Period t

DERIVATION

Reference: Macroeconomics (2nd Edition) by R. Glenn Hubbard and Anthony Patrick O'Brien (Publisher: Pearson)

Standard budget deficit is given by:

$$D_t = i_t B_{t-1} + G_t - T_t$$

To capture the role of seignorage (essentially represents a transfer of wealth from individuals holding money to the government), we extend the above equation to include change in monetary base:

$$D_t = i_t B_{t-1} + G_t - T_t - \Delta M_t$$

Note that the change in government debt is given by:

$$\Delta B_t = B_t - B_{t-1} = D_t = i_t B_{t-1} + G_t - T_t - \Delta M_t$$

Divide both sides by nominal GDP and rearrange to get

$$\frac{B_t}{P_t Y_t} = (1 + i_t)\frac{B_{t-1}}{P_t Y_t} + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Modify pervious equation as follows (multiply and divide the first term on the right hand side by $P_{t-1} Y_{t-1}$):

$$\frac{B_t}{P_t Y_t} = (1 + i_t)\left(\frac{P_{t-1} Y_{t-1}}{P_t Y_t}\right)\left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}}\right) + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Note the following condition,

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) = \frac{P_t}{P_{t-1}}\frac{Y_t}{Y_{t-1}} = (1 + \pi_t)(1 + g_t)$$

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) = (1 + \pi_t)(1 + g_t) \approx 1 + \pi_t + g_t$$

Using the approximation

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) \approx 1 + \pi_t + g_t$$

We get:

$$\frac{B_t}{P_t Y_t} = \left(\frac{1 + i_t}{1 + \pi_t + g_t}\right)\left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}}\right) + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Another useful approximation:

l ÚE •š / ÚE Êš E • š p N ÚE •š F Êš F • š

Sα

nš / | š · ·š L :ÚE •š F Êš F • š; l nš? Ú / | š? Ú · ·š? Ú p E sš / | š · ·š F €š / | š · ·š F ¿t š / | š · ·š

Rearrange terms to get: